Resonant and Secular Families of the Kuiper Belt

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Abstract. We review ongoing efforts to identify occupants of mean-motion resonances (MMRs) and collisional families in the Edgeworth-Kuiper belt. Direct integrations of trajectories of Kuiper belt objects (KBOs) reveal the 1:1 (Trojan), 5:4, 4:3, 3:2 (Plutino), 5:3, 7:4, 9:5, 2:1 (Twotino), and 5:2 MMRs to be inhabited. Apart from the Trojan, resonant KBOs typically have large orbital eccentricities and inclinations. The observed pattern of resonance occupation is consistent with resonant capture and adiabatic excitation by a migratory Neptune; however, the dynamically cold initial conditions prior to resonance sweeping that are typically assumed by migration simulations are probably inadequate. Given the dynamically hot residents of the 5:2 MMR and the substantial inclinations observed in all exterior MMRs, a fraction of the primordial belt was likely dynamically pre-heated prior to resonance sweeping. A pre-heated population may have arisen as Neptune gravitationally scattered objects into trans-Neptunian space. The spatial distribution of Twotinos offers a unique diagnostic of Neptune's migration history. The Neptunian Trojan population may rival the Jovian Trojan population, and the former's existence is argued to rule out violent orbital histories for Neptune. Finally, lowest-order secular theory is applied to several hundred non-resonant KBOs with well-measured orbits to update proposals of collisional families. No convincing family is detected.

1. Introduction

Plutinos are Kuiper belt objects (KBOs) that occupy the exterior 3:2 mean-motion resonance (MMR) established by Neptune (see, e.g., Jewitt & Luu, 2000). The preponderance of Plutinos having large orbital eccentricities has been interpreted to imply that Neptune's orbit expanded outwards by several AUs over a timescale of $\tau \gtrsim 10^6$ yr (Malhotra, 1995). The expansion was supposedly driven by angular momentum exchange with ancient planetesimals interspersed among the giant planets and having about as much mass as the ice giants (Fernandez & Ip, 1984; Hahn & Malhotra, 1999; Gomes, 2003). As Neptune spiralled outwards, its exterior MMRs swept across the primordial Kuiper belt, captured KBOs, and amplified their orbital eccentricities.

Simulations for which Neptune's outward migration is smoothly monotonic and for which $\tau \gtrsim 10^7$ yr predict the 2:1 MMR to be populated with roughly as many objects as the 3:2 MMR, for reasonable assumptions regarding the distribution of orbital elements prior to resonance sweeping (Malhotra, Duncan, & Levison, 2000; Chiang & Jordan, 2002). Initial discoveries of Plutinos but not of "Twotinos" (2:1 resonant KBOs) led to speculation that $\tau < 10^6$ yr; the strength of the 2:1 MMR is weaker than that of the 3:2, and the former resonance's capture efficiency decreases more rapidly with increasing migration rate (Ida et al., 2000; Friedland, 2001). Reports of the absence of Twotinos proved greatly exaggerated; wide-field surveys for KBOs and painstaking astrometric recovery observations world-wide have now secured ~200 KBO orbits with sufficient accuracy that ~7 Twotinos are confidently identified (Chiang & Jordan, 2002; Chiang et al., 2003, hereafter C03; and see Table I of the present paper). A plethora of other resonances are also observed to be occupied; what these other resonances imply about the dynamical history of Neptune and the Kuiper belt is summarized herein.

Following Hirayama (1918), we ask also whether certain KBOs trace their lineage to parent bodies that experienced catastrophic, collisional disruption. The proportion of KBOs that are shattered fragments records the collisonal history of the belt and constrains its mass as a function of time. Candidate collisional families are identified by similarities in their observed spectra and in their so-called "proper" or "free" orbital elements. Here we present a first-cut calculation of the free elements of KBOs with accurately measured orbits.

In §2, we describe our procedure for rigorously identifying resonant KBOs. In §3, we highlight the implications of 3 occupied resonances—the 2:1, 5:2, and 1:1 MMRs—for the dynamical history of the outer solar system. In §4, we present the free orbital elements of 227 non-resonant KBOs and attempt to identify a candidate collisional family.

2. Resonance Identification

By definition, a mean-motion resonant KBO is characterized by one or more resonant arguments that librate, i.e., undergo bounded oscillations with time. Each resonant argument takes the form

$$\phi_{p,q,m,n,r,s} = p\lambda - q\lambda_N - m\tilde{\omega} - n\Omega - r\tilde{\omega}_N - s\Omega_N \ , \qquad (1)$$

where λ, $\tilde{\omega}$, and Ω are the mean longitude, longitude of pericenter, and longitude of ascending node of the KBO, respectively. Those same

quantities subscripted by "N" are those of Neptune, and p, q, m, n, r, and s are integers. By rotational invariance, $p - q - m - n - r - s = 0$.

Identifying resonant objects is a straightforward matter of integrating forward the trajectories of Neptune and of the KBOs and examining the behavior of $\phi_{p,q,m,n,r,s}$ for every object. Our present implementation tests for libration of 107 different values of $\{p, q, m, n, r\}$; for convenience, and because resonances associated with the small inclination of Neptune are weak, we set $s = 0$.[1] The integrations are carried out with the SWIFT software package (swift_rmvs3), developed by Levison & Duncan (1994) and based on the N-body map of Wisdom & Holman (1991). We include the influence of the four giant planets, treat each KBO as a massless test particle, and integrate trajectories forward for 3 Myr using a timestep of 50 days, starting at Julian date 2451545.0. Any duration of integration longer than the mean-motion resonant libration period, $\sim 10^4$ yr, would be adequate to test for resonance membership. However, we have found by numerical experiment that adopting durations less than $\sim$1 Myr yields membership in a host—often, more than 5—weak resonances for a given object. Upon integrating for longer durations, many objects escape most of these high-order resonances. Since we are interested in long-term, presumably primordial residents of resonances, we integrate for as long as is computationally practical, i.e., 3 Myr. In cases of particular interest—e.g., the Neptune Trojan—we integrate trajectories up to 1 Gyr to test for long-term stability.

Initial positions and velocities of 407 objects are computed using the formalism of Bernstein & Khushalani (2000) in the case of short-arc orbits, and from E. Bowell's database in the case of long-arc orbits. These data are maintained and continuously updated at Lowell Observatory; we report here results obtained using orbit solutions calculated on Jan 2 2003. About half of these 407 objects were discovered by the Deep Ecliptic Survey (Millis et al., 2002; C03; Elliot et al., 2003). For every object, we integrate forward the best-fit orbit solution in addition to 2 "clones": orbit solutions that lie on the 3σ confidence surface and that are characterized by maximum and minimum semi-major axes. For the cloned solutions, the other 5 orbital elements are adjusted according to their correlation with semi-major axis. Our rationale for singling out semi-major axis is explained in C03.

An object is considered "securely resonant" if all three sets of initial conditions yield libration of the same resonant argument(s) for the entire duration of the integration. Of 407 KBOs tested, we find 75 to be securely resonant. Not all of the 407 KBOs have orbital parameters

[1] We also test for membership in the secular Kozai resonance, in which the argument of perihelion, ω, librates. However, the 3-Myr duration of our integrations is marginally too short to witness a full Kozai libration cycle for several objects.

Table I. Resonant 3σ-confident KBOs

Resonance	Name
1:1	2001QR$_{322}$
5:4	1999CP$_{133}$, 2002GW$_{32}$
4:3	1998UU$_{43}$, 2000CQ$_{104}$, (15836) 1995DA$_{2}$
3:2	(28978) Ixion, 1998UR$_{43}$, 1998WS$_{31}$, 1998WU$_{31}$, 1998WV$_{31}$, 1998WW$_{24}$, 1998WZ$_{31}$, 2000CK$_{105}$, 2001KY$_{76}$, 2001KB$_{77}$, 2001KD$_{77}$, 2001QF$_{298}$, 2001QG$_{298}$, 2001RU$_{143}$, 2001RX$_{143}$, (15788) 1993SB, (15789) 1993SC, (15810) 1994JR$_{1}$, (15820) 1994TB, (15875) 1996TP$_{66}$, (19299) 1996SZ$_{4}$, (20108) 1995QZ$_{9}$, (24952) 1997QJ$_{4}$, (32929) 1995QY$_{9}$, (33340) 1998VG$_{44}$, (38628) 2000EB$_{173}$, (47171) 1999TC$_{36}$, (47932) 2000GN$_{171}$, (55638) 2002VE$_{95}$, 1993RO, 1995HM$_{5}$, 1996RR$_{20}$, 1996TQ$_{66}$, 1998HH$_{151}$, 1998HK$_{151}$, 1998HQ$_{151}$, 1999CE$_{119}$, 1999CM$_{158}$, 1999TR$_{11}$, 2000FV$_{53}$, 2000GE$_{147}$, 2001FL$_{194}$, 2001FR$_{185}$, 2001VN$_{71}$, 2001YJ$_{140}$, 2001FU$_{172}$, 2002GY$_{32}$
5:3	(15809) 1994JS, 1999CX$_{131}$, 2001XP$_{254}$, 2001YH$_{140}$
7:4	2000OP$_{67}$, 2001KP$_{77}$, 1999KR$_{18}$, 2000OY$_{51}$
9:5	2001KL$_{76}$
2:1	2000QL$_{251}$, (20161) 1996TR$_{66}$, (26308) 1998SM$_{165}$, 1997SZ$_{10}$, 1999RB$_{216}$, 2000JG$_{81}$, 2001FQ$_{185}$
5:2	(38084) 1999HB$_{12}$, 2001KC$_{77}$, (26375) 1999DE$_{9}$, 2000FE$_{8}$, 2001XQ$_{254}$, 2002GP$_{32}$

measured with sufficient accuracy to permit meaningful classification and more than our 75 identified objects may well be resonant; a more careful assessment of the resonant vs. non-resonant population ratio is deferred to Elliot et al. (2003). Orbital elements of securely resonant KBOs are plotted in Figure 1. Occupied resonances include the 1:1 (Trojan), 5:4, 4:3, 3:2, 5:3, 7:4, 9:5, 2:1, and 5:2 MMRs. They are all of eccentricity-type, though occasionally objects inhabit both eccentricity-type and inclination-type resonances. A histogram of resonance occupation, listing all resonances that we test for, is displayed in Figure 2. Names of the 75 resonant KBOs are listed in Table I. The results presented in Figures 1 and 2 and Table 1 differ slightly from those presented in C03, because here we have made our test for resonance membership more stringent by adopting a 3σ criterion rather than a 1σ criterion; only a handful of objects ($\sim$10) are found in C03 and not the present work.

Nearly all of the resonances that are observed to be occupied are predicted to be occupied by standard, smooth migration simulations

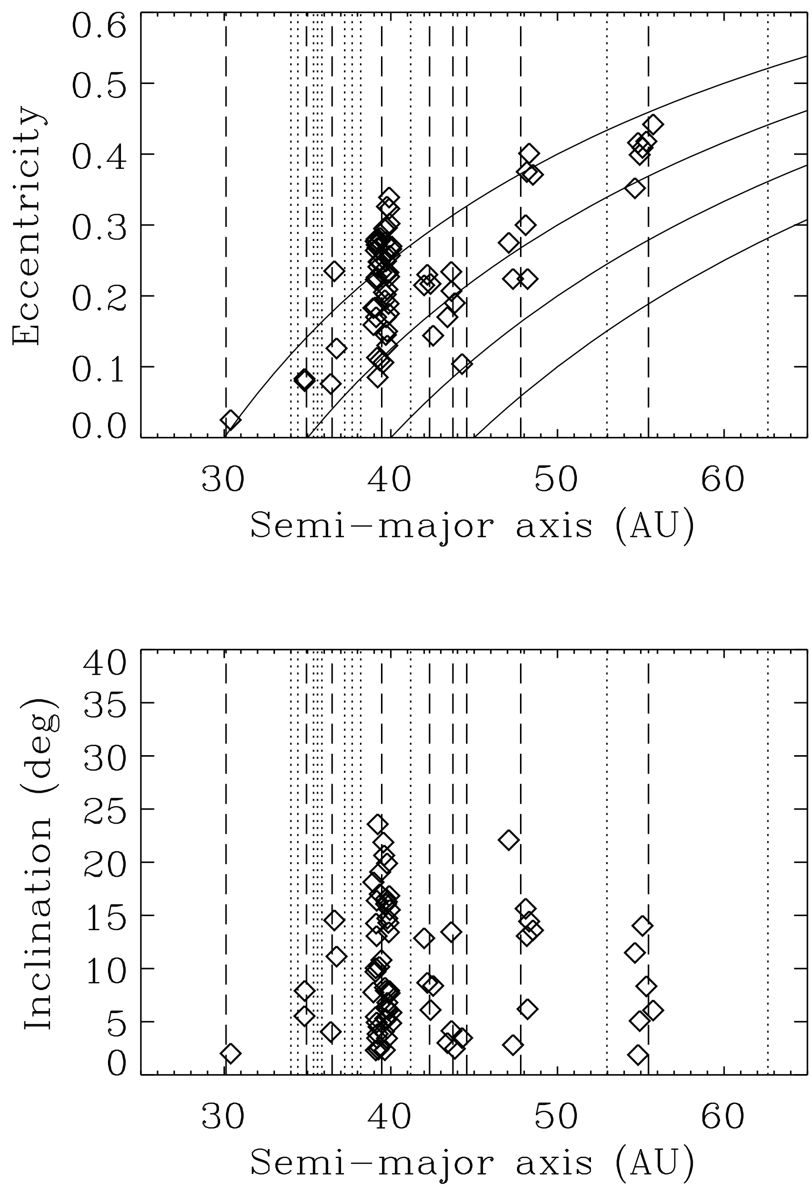


Figure 1. Eccentricities, inclinations, and semi-major axes of 75 securely resonant KBOs. Dotted lines indicate locations of nominal resonance with Neptune; dashed lines indicate occupied resonances. In order of increasing semi-major axis, the occupied resonances include the 1:1 (Trojan), 5:4, 4:3, 3:2, 5:3, 7:4, 9:5, 2:1, and 5:2 MMRs. Elements are heliocentric, referred to the J2000 ecliptic plane, and evaluated at epoch 2451545.0 JD. Uncertainties are too small to plot here.

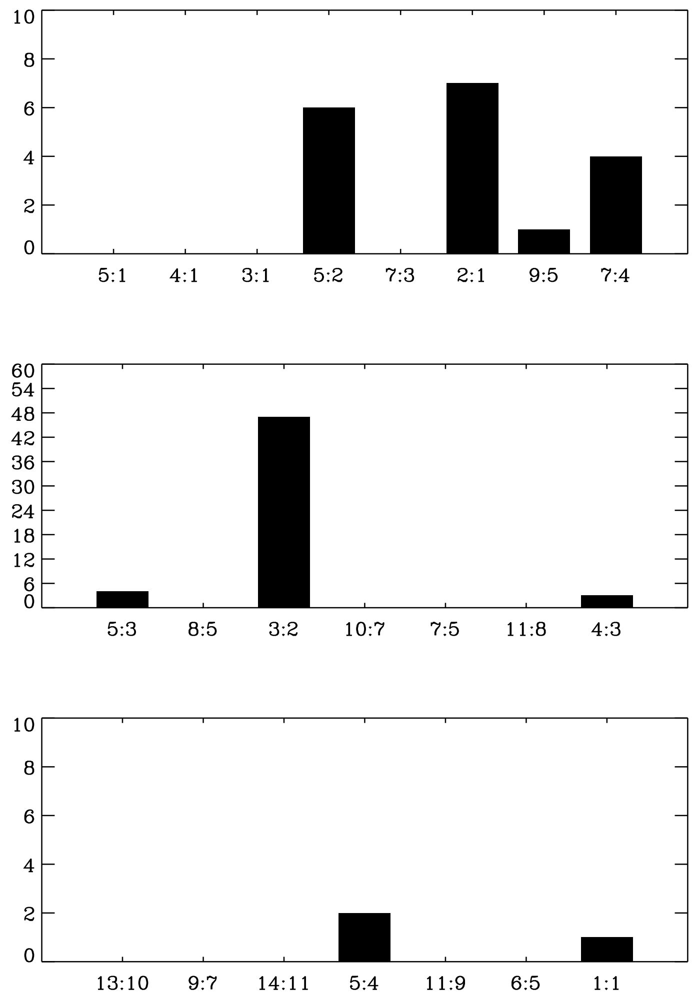


Figure 2. Number of KBOs occupying a given resonance.

for which $\tau \gtrsim 10^6$ yr. Noteworthy exceptions include the (innermost) 1:1 and (outermost) 5:2 MMRs. We turn our attention to these extreme resonances, in addition to the newly discovered class of (2:1) Twotinos, to examine their cosmogonic implications.

3. Three Resonances of Interest

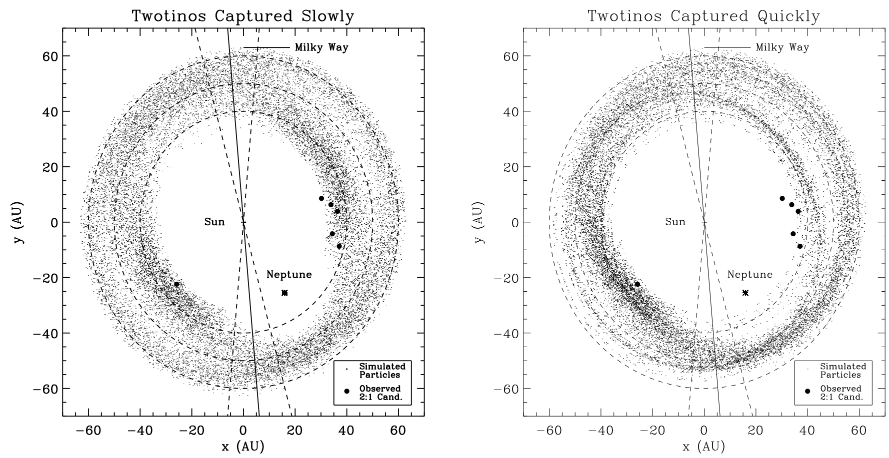
3.1. The 5:2 MMR

This resonance is difficult to populate under standard migration scenarios that presume cold initial conditions.[2] C03 report that if $\tau \sim 10^7$ yr, and if the initial eccentricities and inclinations of KBOs prior to resonance sweeping are low ($e_{init}, i_{init} \lesssim 0.05$), then the final, post-sweeping population ratio between the 5:2 and 2:1 resonant objects is of order 1-to-90. This ratio conflicts with the observed ratio of ∼6-to-7. Moreover, the simulation predicts final eccentricities of 5:2 resonant KBOs of $e \sim 0.2$ and final inclinations of $i \lesssim 1°$, values too small compared with those observed. Indeed, observed occupants of the 5:2 MMR hold the record among resonant objects for the highest eccentricities (up to $e \sim 0.45$), and they sport high inclinations (up to $i \sim 15°$) as well.

As demonstrated by C03, observations of 5:2 resonant KBOs may still be reconciled with the migration hypothesis if one presumes hot initial conditions prior to resonance sweeping. C03 find numerically that the sweeping 5:2 MMR more easily captures KBOs that already possess eccentricities and inclinations of order 0.2 prior to resonance encounter. (Of course, by presupposing such highly excited orbits, the need for any planetary migration at all becomes less pressing!) The abundance of hot particles in the 5:2 MMR, together with the large orbital inclinations observed across the entire belt (both in and out of resonances) and the existence of high-perihelion objects such as 2000CR$_{105}$ (Millis et al., 2002; Gladman et al., 2002), clearly point to at least one other heating mechanism apart from adiabatic excitation by slowly sweeping MMRs.

What might have caused this pre-heating? We are aware of two proposals. Thommes, Duncan, & Levison (2002) propose that the embryonic cores of Neptune and Uranus, both of mass ∼10 $M_\oplus$, were scattered by Jupiter and Saturn into the ancient belt and heated KBOs by dynamical friction. Alternatively, Gomes (2003) points out that under the classic migration scenario, planetesimals should have undergone

[2] This discussion is subject to the caveat that only a fraction of the observed 5:2 resonant KBOs may be primordial, long-term residents. C03 undertake a Gyr-long integration of three observed 5:2 resonant KBOs and conclude that at least two (38084 and 2001KC$_{77}$) can be primordial residents with moderate libration amplitudes $[\max(\phi) - 180°]$ of less than 100°. The third KBO (1998WA$_{31}$) departs the resonance after 3 Myr; it may therefore be a scattered-disk object that has only temporarily stuck to the 5:2 MMR. Based on this preliminary study, it seems clear that at least some of the observed objects are primordial, and to the extent that objects 38084 and 2001KC$_{77}$ have more precisely measured orbits than 1998WA$_{31}$, the primordial subset may grow as the astrometry improves.



a. $\tau = 10^7$ yr   b. $\tau = 10^6$ yr

Figure 3. Predicted snapshots, viewed from the ecliptic pole, of the spatial distribution of 2:1 resonant Kuiper belt objects. In the left panel, Twotinos were captured into resonance by Neptune as that planet migrated outwards into the Kuiper Belt over a timescale of 10^7 yr. In the right panel, the migration timescale is 10^6 yr. Whether the ancient outward migration of Neptune was slow or fast has dramatic consequences for the longitudinal distribution of Twotinos. Positions of several recently discovered Twotinos are marked by solid circles; they are too few to test these ideas. Dashed circles correspond to heliocentric radii of 40, 50, and 60 AU, and radial lines indicate the position of the Galactic plane, $\pm 10°$ Galactic latitude.

close encounters with Neptune that propelled them onto orbits having larger semi-major axes, eccentricities, and inclinations, and that these scattered planetesimals were subsequently swept over by mean-motion resonances. Our discovery of the first Neptune Trojan (C03) leads us to favor the mechanism of Gomes (2003), as we discuss in §3.3.

3.2. The 2:1 MMR

Twotinos furnish a diagnostic of planetary migration (Chiang & Jordan, 2002). The 2:1 resonance allows for asymmetric libration; at large KBO eccentricities, $\phi_{2,1,1,0,0,0}$ ceases to librate about 180°, and instead librates about angles in the vicinity of $\pm 70°$ (Beauge, 1994). Whether a KBO is swept into libration about $\langle\phi\rangle \approx 70°$ or into libration about $\langle\phi\rangle \approx -70°$ depends on the migration timescale. If $\tau \approx 1$–3×10^6 yr, three times as many KBOs librate about the latter angle than about the former. The magnitude of the asymmetry monotonically decreases with increasing τ, and nearly vanishes if $\tau \gtrsim 10^7$ yr.

The asymmetry in libration center populations translates directly into an asymmetry on the sky, as illustrated in Figure 3. If $\tau < 10^7$

yr, more Twotinos should be seen coming into perihelion at longitudes lagging Neptune's instantaneous longitude than at longitudes leading it. Observed numbers of Twotinos are too low to test this prediction. If such an asymmetry were to be observed in the future (say, with the PAN-STARRS observatory), it would constitute strong evidence supporting planetary migration. If the distribution of Twotinos is found to be symmetric with respect to the Sun-Neptune line, then such a finding would be consistent with the planetary migration hypothesis and would force $\tau > 10^7$ yr.[3]

3.3. The 1:1 MMR

The first known Neptune Trojan, 2001QR$_{322}$, was discovered by the Deep Ecliptic Survey (C03). The object can librate about Neptune's forward Lagrange point (L4) in a tadpole-type trajectory for at least 1 Gyr (C03). The osculating, heliocentric, and J2000 ecliptic-based eccentricity and inclination are small, of order 0.03. The libration center is $\langle\phi_{1,1,0,0,0,0}\rangle \approx 65^\circ$, the libration amplitude is $\Delta\phi \equiv \max(\phi) - \langle\phi\rangle \approx 24^\circ$, and the libration period is $T \approx 10^4$ yr.[4] For an albedo of 12–4%, the diameter of 2001QR$_{322}$ is 130–230 km. Based on the area of sky observed to date by the Deep Ecliptic Survey and various assumed distributions of orbital elements of Neptune Trojans (see Nesvorny & Dones, 2002), the total number of Neptune Trojans resembling 2001QR$_{322}$ ranges between 20 and 60. Such a population would be comparable to that of Jupiter's Trojans, for which $\sim$10 exist having diameters between 100 and 200 km (Davis et al. 2003).

Trojans probably do not owe their existence to planetary migration; the overwhelming fate of particles that cross Neptune's orbit is to be scattered onto orbits having high eccentricities, high inclinations, and semi-major axes substantially different from Neptune's (C03). One probable step in the process of accruing Trojans is substantial mass accretion by the host planet. If the mass of the host planet grows on a timescale longer than the Trojan libration period, libration amplitudes of test particles loosely bound to co-orbital resonances shrink; the planet effectively tightens its grip as its mass increases. Horseshoe-type orbits shrink to tadpole-type orbits (Marzari & Scholl, 1998), and libration amplitudes of tadpole-type orbits further decrease with increasing mass, M, of the host planet as

[3] The third and last possibility—that leading Twotinos outnumber lagging Twotinos—is not predicted at all by the migration model. If such an observation were to come to pass, theorists would be forced back to the drawing board.

[4] The computed libration center is offset from the true stable point of 60° because tadpole trajectories are not symmetric about the Lagrange point.

$$\Delta\phi \propto M^{-1/4} \tag{2}$$

(Fleming & Hamilton, 2000). The weakness of the dependence of $\Delta\phi$ on M argues that tightening of Trojan orbits occurred while the host planet accreted the lion's share of its mass. Thus, we are led to the following picture for Neptune's formation and orbital evolution. Neptune accreted the overwhelming bulk of its mass near a heliocentric distance of ~23 AU on a nearly circular orbit and, in so doing, captured a Trojan population by adiabatically securing its hold on whatever co-orbital planetesimals were present. Subsequent slow migration of Neptune and the other giant planets whittled down but did not eliminate Neptune's Trojan population; Gomes (1998) and Kortenkamp, Malhotra, & Michtchenko (2003) find that standard planetary migration scenarios reduce the number of Neptune Trojans to a fraction of order 10% of their original population. The orbital elements of surviving Trojans resembles that of long-term stable Trojans as delineated by Nesvorny & Dones (2002).

The above picture in which Neptune forms as the solar system's outermost giant planet core, and in which it never occupies a substantially eccentric orbit, conflicts with that of Thommes, Duncan, & Levison (2002). In their view, the bulk (~50%) of Neptune is assembled between Jupiter and Saturn; proto-Neptune is subsequently gravitationally scattered onto a highly eccentric orbit that takes it into the Kuiper belt. Its trajectory then circularizes as a consequence of dynamical friction with planetesimals. We do not understand how Neptune can capture and retain a retinue of Trojans as it careens back and forth across the solar system.

4. Collisional Families

We turn now to non-resonant KBOs and ask whether some objects are collisional fragments based on their orbital elements. We follow Hirayama (1918) and compute the free eccentricities and free inclinations of KBOs. Objects sharing similar values of the free elements are deemed members of a candidate collisional family. To extract the free elements, we employ the secular theory of Brouwer & van Woerkom (1950) to subtract the forced elements from the observed osculating elements. The procedure is identical to that described by Chiang (2002); here we update that work by increasing our sample size to 227 non-resonant KBOs whose fractional 3σ uncertainties in semi-major axis are less than 6%, as estimated using the methodology of Bernstein & Khushalani (2000).

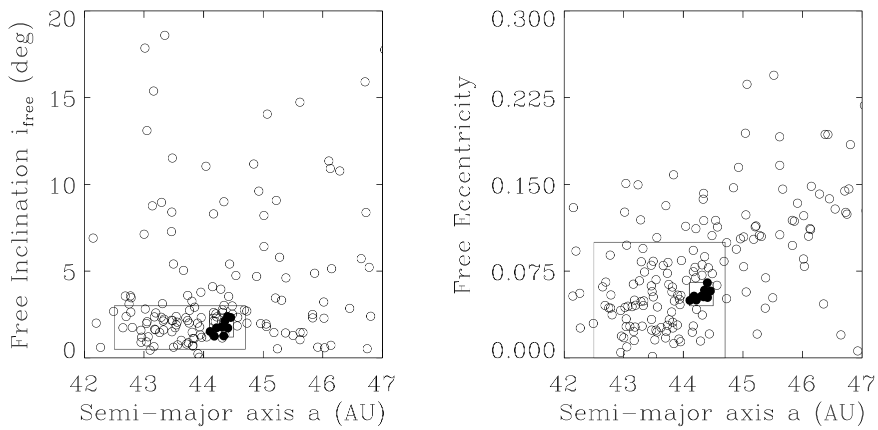


Figure 4. Free inclinations, free eccentricities, and osculating semi-major axes of non-resonant KBOs. Solid circles, enclosed within a small box, mark 9 members of a candidate collisional family. A larger box is (somewhat arbitrarily) drawn around the volume in which points seem to be distributed uniformly and encloses 71 points. If points are distributed randomly within this larger volume according to a uniform probability distribution, the probability of 9 points out of 71 lying within the smaller volume is remarkably small, $\sim 10^{-6}$. Unfortunately, the velocity dispersion of the cluster is too small compared to the escape velocity of the hypothesized parent body; the cluster of points probably does not correspond to a real collisional family.

Figure 4 displays the free eccentricities, free inclinations, and osculating semi-major axes (which are constants of the motion in secular theory) of our sample. Nine KBOs are highlighted that appear, by eye, to be strongly clustered in ($e_{\rm free}$, $i_{\rm free}$, a) space. A box having dimensions that can just enclose these nine points, if placed anywhere else in ($e_{\rm free}$, $i_{\rm free}$, a) space, encloses fewer than nine points. We list the properties of the nine KBOs in Table II. Are these nine KBOs fragments of a once disrupted parent body? The short answer is, probably not. In what follows, we describe our efforts at determining the significance of this clump of points. We offer arguments for and against the reality of this candidate family, partly to illustrate the difficulties involved in identifying real families.

Our candidate family is similar to the one proposed by Chiang (2002); indeed, three members are shared between them (1998HM$_{151}$, 1999RC$_{215}$, and 2000PY$_{29}$). We regard our candidate family to supersede that proposed by Chiang (2002), since our dataset is larger and more current. Note that unlike the family originally proposed by Chiang (2002), which clusters only in a and $i_{\rm free}$, our candidate family

Table II. Nine Clustered KBOs

Name	a (AU)	$e_{\rm free}$	$i_{\rm free}$ (deg)	H_V (mag)
(52747) 1998HM$_{151}$	44.18	0.053	1.25	7.9
1999OA$_4$	44.45	0.058	2.33	7.9
1999RC$_{215}$	44.40	0.065	2.38	6.9
2000PM$_{30}$	44.11	0.050	1.52	7.9
2000PW$_{29}$	44.22	0.050	1.73	8.2
2000PY$_{29}$	44.34	0.053	1.26	7.1
2000YA$_2$	44.41	0.052	1.72	6.9
2001QS$_{322}$	44.31	0.054	1.78	5.7
2001QZ$_{297}$	44.36	0.059	2.13	6.3

clusters in all three dimensions. Moreover, the greater size of our sample now makes clear that not all of the KBOs in the range of semi-major axes spanned by our family are probably members of the same family; additional, less clustered objects exist at large inclinations and a variety of eccentricities. This feature lends further support to the reality of our proposed family. If the objects in Table II do constitute fragments of the same parent body, the minimum diameter of the parent body would be 700 km, based on the measured H_V's and an assumed albedo of 5%.

We perform three tests to assess the statistical and physical significance of our candidate family. The candidate passes the first test, but fails the other two.

The first *ad hoc* and crude estimate of the statistical significance of this cluster proceeds as follows. As shown in Figure 4, we draw a large box that encloses a volume in which points appear to be distributed uniformly. There are 71 points within this large box. Within this volume we draw another, smaller box that encloses the 9 KBOs. We then ask, if we randomly distribute 71 points in the larger box according to a uniform probability distribution, what is the probability that 9 points out of 71 land within the smaller box? The answer is $\sim 10^{-6}$, a number that we regard to be sufficiently small to warrant further investigation.

A second test, suggested to us by Renu Malhotra, asks whether the dispersion of "free velocities" exhibited by candidate family members matches the expected dispersion from a catastrophic collision. A minimum estimate for the latter is the escape velocity of the parent body; for our putative parent body of minimum diameter 700 km, the escape velocity is at least $\sim$0.4 km/s. Immediately after the hypothesized collision, fragments must have been moving relative to each other with velocities near or above this value to avoid gravitational re-

accumulation. We estimate the actual velocity dispersion by calculating the standard deviation of $\sqrt{e_{\rm free}^2 + i_{\rm free}^2} v_K$, where $v_K \approx 5$ km/s is the Keplerian orbital velocity of family members. The answer is 0.03 km/s $\ll$ 0.4 km/s. This finding casts doubt on the reality of our proposed family. However, adding more objects at greater values of $e_{\rm free}$ and $i_{\rm free}$ to our candidate family would help to reconcile the velocity dispersions.

A third test, suggested to us by Brad Hansen, employs Ward's minimum-variance method for quantitatively identifying clusters in data sets (Murtagh & Heck, 1987). This method agglomerates objects in order of increasing separation in ($e_{\rm free}$, $i_{\rm free}$, a) space. A convincing segregation would demand the nine candidate family members to be agglomerated consecutively together and the distance between this agglomeration and others to be large. Unfortunately, not only were the nine members not agglomerated consecutively together, but no single agglomeration of objects emerged that was clearly distinguishable from the remaining data set.

We conclude that no rigorously defensible collisional family can be identified among the 227 non-resonant KBOs tested. The tests served to highlight the subjective nature of identifying families. Despite the difficulties involved, we emphasize that a definitive measurement of the proportion of KBOs that are shattered fragments would offer direct insight into the belt's mass and velocity dispersion as a function of time. If recent proposals regarding the formation of KBO binaries are correct, so that nearly all KBOs form as nearly equal-mass binaries (Goldreich, Lithwick, & Sari 2003), then KBOs that are found today not to be binary would comprise the shattered population.

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